

Lepton-Specific Two-Higgs Doublet Model: Experimental Constraints and Implication on Higgs Phenomenology

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Abstract

We examine various direct and indirect constraints on the lepton-specific two-Higgs doublet model and scrutinize the property of the Higgs bosons in the allowed parameter space. These constraints come from the precision electroweak data, the direct search for Higgs boson, the muon anomalous magnetic moment, as well as some theoretical consistency requirements. We find that in the allowed parameter space the CP-odd Higgs boson A is rather light ($m_A < 30$ GeV with 95% possibility), which is composed dominantly by the leptonic Higgs and decays dominantly into $\tau^+\tau^-$; while the SM-like Higgs boson h (responsible largely for electroweak symmetry breaking) decays dominantly in the mode $h \rightarrow AA \rightarrow 4\tau$ with a large decay width, which will make the Higgs discovery more difficult at the LHC. Whereas, this scenario predicts a branching ratio $Br(Z \rightarrow \tau^+\tau^- A)$ ranging from 10^{-5} to 10^{-4} , which may be accessible at the the GigaZ option of the ILC.

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I. INTRODUCTION

The phenomenological success of the standard model (SM) has significantly limited the possibility of new physics except for the Higgs sector which remains untested. There are numerous speculations on the possible extensions of the Higgs sector, among which the simplest is to introduce one more Higgs doublet. Compared with the SM, such simple two-Higgs doublet models usually have much more complicated Higgs phenomenology. In the SM a single Higgs doublet is responsible for the electroweak symmetry breaking and the Higgs couplings with fermions and gauge bosons are completely determined by their masses, and therefore there is little guesswork in determining the discovery channels for the Higgs boson [1]. In the two-Higgs doublet models, however, the addition of new scalars and the modification of the Higgs interactions will significantly complicate the Higgs discovery at the LHC [2]. Given the imminent running of the LHC, the phenomenological study of various such models is urgently important.

In this paper we focus on a special two-Higgs doublet model called the lepton-specific two-Higgs doublet model (L2HDM) [3]. Since this model is arguably well motivated from some fundamental theory and also has some phenomenological virtues (e.g., it can provide a natural explanation for the leptonic cosmic ray signals reported by PAMELA and ATIC [4]), it has attracted much attention [5, 6]. We will check various constraints on the model parameters and then scrutinize the property of the Higgs bosons in the allowed parameter space. These constraints come from the precision electroweak data, the direct search for Higgs boson, the muon anomalous magnetic moment, as well as some theoretical consistency requirements. Our main observation is that in the allowed parameter space the CP-odd Higgs boson A must be light ($m_A < 30$ GeV with 95% possibility), which is composed dominantly by the leptonic Higgs and decays dominantly into $\tau\bar{\tau}$; while the SM-like Higgs boson h (responsible largely for electroweak symmetry breaking) decays dominantly in the mode $h \rightarrow AA \rightarrow 4\tau$ with a decay width usually exceeding several tens of GeV, which may make the Higgs discovery more difficult at the LHC.

This paper is organized as follows. In Sec. II we recapitulate the L2HDM model. In Sec. III we examine various constraints on the parameter space and study the properties of the Higgs bosons in the allowed parameter space. Finally, In Sec. IV we give our conclusion.

II. THE LEPTON-SPECIFIC TWO-HIGGS DOUBLET MODEL

The L2HDM is a special two-Higgs doublet model in which one Higgs doublet ϕ_1 couples only to leptons while the other doublet ϕ_2 couples only to quarks. Both Higgs doublets contribute to the electroweak symmetry breaking: $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ with v_1 and v_2 being respectively the vacuum expectation values of ϕ_1 and ϕ_2 ; whereas their relative contributions can be quite different and can be parameterized by the ratio $\tan\beta = v_2/v_1$. So for a large $\tan\beta$ the lepton Yukawa couplings can be greatly enhanced.

The Yukawa interactions and the Higgs potential are given by [6]

$$\mathcal{L}_Y = -Y_e^{ij} \bar{\ell}_i \phi_1 e_j - Y_u^{ij} \bar{q}_i \phi_2^c u_j - Y_d^{ij} \bar{q}_i \phi_2 d_j + h.c. \quad (1)$$

$$\begin{aligned} V = & m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 - \left(m_3^2 \phi_1^\dagger \phi_2 + h.c. \right) + \frac{\lambda_1}{2} |\phi_1|^4 + \frac{\lambda_2}{2} |\phi_2|^4 \\ & + \lambda_3 |\phi_1|^2 |\phi_2|^2 + \lambda_4 |\phi_1^\dagger \phi_2|^2 + \frac{\lambda_5}{2} \left[(\phi_1^\dagger \phi_2)^2 + h.c. \right], \end{aligned} \quad (2)$$

where i, j are generation indices, Y_e , Y_u and Y_d are 3×3 Yukawa matrices, q_i and ℓ_i denote respectively the the left-handed quark and lepton fields, u_i and d_i denote respectively the right-handed up- and down-type quark fields, e_i denotes the right-handed lepton fields, and m^2 and λ are free parameters.

Just like the usual two-Higgs doublet model [7], the spectrum of the Higgs sector includes three massless Goldstone modes, which become the longitudinal modes of W^\pm and Z bosons, and five massive physical states: two CP -even states h and H , a pseudoscalar A , and a pair of charged states H^\pm . These states are related to the doublets ϕ_1 and ϕ_2 by

$$\phi_1^0 = \frac{1}{\sqrt{2}} (v_1 + H \cos\alpha - h \sin\alpha + iG^0 \cos\beta - iA \sin\beta), \quad (3)$$

$$\phi_2^0 = \frac{1}{\sqrt{2}} (v_2 + H \sin\alpha + h \cos\alpha + iG^0 \sin\beta + iA \cos\beta), \quad (4)$$

$$\phi_1^\pm = G^\pm \cos\beta - H^\pm \sin\beta, \quad (5)$$

$$\phi_2^\pm = G^\pm \sin\beta + H^\pm \cos\beta, \quad (6)$$

where α is the mixing angle that diagonalizes the mass matrix of the CP -even Higgs fields.

Due to the constraint $v_1^2 + v_2^2 = (246 \text{ GeV})^2$, the eight free parameters in Eq. (2), i.e., λ_i ($i = 1, \dots, 5$) and m_i^2 ($i=1,2,3$), reduce to seven. In our analysis we choose the following seven parameters as the input parameters of the L2HDM:

$$m_h, m_H, m_A, m_{H^\pm}, \tan\beta, \sin\alpha, \lambda_5, \quad (7)$$

where m_h , m_A , m_H , and m_{H^\pm} are the masses of the corresponding physical states. Throughout this paper, we use H (h) to denote the Higgs boson with ϕ_1^0 (ϕ_2^0) as its dominant component, which means that we choose $\cos^2 \alpha > 1/2$.

The interactions of the Higgs physical states with fermions are then given by [6]

$$\begin{aligned}
\mathcal{L}_Y = & -\frac{gm_{e_i}}{2m_W \cos \beta} (\cos \alpha \bar{e}_i e_i H - \sin \alpha \bar{e}_i e_i h) \\
& -\frac{gm_{q_i}}{2m_W \sin \beta} (\sin \alpha \bar{q}_i q_i H + \cos \alpha \bar{q}_i q_i h) \\
& +\frac{igm_{u_i}}{2m_W} \cot \beta \bar{u}_i \gamma_5 u_i A - \frac{igm_{d_i}}{2m_W} \cot \beta \bar{d}_i \gamma_5 d_i A \\
& +\frac{igm_{e_i}}{2m_W} \tan \beta \bar{e}_i \gamma_5 e_i A \\
& +\frac{gV_{ij}}{\sqrt{2}m_W} \cot \beta \bar{u}_i (m_{u_i} P_L - m_{d_j} P_R) d_j H^+ \\
& +\frac{gm_{e_i}}{\sqrt{2}m_W} \tan \beta \bar{\nu}_i P_R e_i H^+.
\end{aligned} \tag{8}$$

Obviously, for a large $\tan \beta$ the lepton Yukawa couplings are greatly enhanced relative to the SM prediction. One can also check that the couplings of ZZh and ZZH are given by

$$V_{ZZh} = \frac{gm_Z}{\cos \theta_W} \sin(\beta - \alpha), \tag{9}$$

$$V_{ZZH} = \frac{gm_Z}{\cos \theta_W} \cos(\beta - \alpha), \tag{10}$$

which satisfy the sum rule $V_{ZZh}^2 + V_{ZZH}^2 = V_{ZZh_{SM}}^2$. For a large $\tan \beta$ (this is the case required by the experimental constraints, as shown below), the coupling ZZh is dominant over ZZH , so h is usually called the SM-like Higgs boson.

III. CONSTRAINTS ON THE L2HDM

We note that both the theoretical consistency and the electroweak data have limited the parameter space of the L2HDM. In our study we consider the following theoretical constraints:

- (1) The perturbativity is valid in the Higgs sector, which requires $\lambda_i < 4\pi$ ($i = 1, \dots, 5$).
- (2) The S -matrix satisfies all relevant tree-unitarity constraints, which implies that the

quartic couplings λ_i satisfy [8]

$$\begin{aligned}
3(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + 4(2\lambda_3 + \lambda_4)^2} &< 16\pi, \\
\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4|\lambda_5|^2} &< 16\pi, \\
\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4|\lambda_5|^2} &< 16\pi, \\
\lambda_3 + 2\lambda_4 \pm 3|\lambda_5| &< 8\pi, \\
\lambda_3 \pm \lambda_4 &< 8\pi, \\
\lambda_3 \pm |\lambda_5| &< 8\pi.
\end{aligned} \tag{11}$$

(3) The scalar potential in Eq. (2) is finite at large field values and contains no flat directions, which translate into the bounds [7]

$$\begin{aligned}
\lambda_{1,2} &> 0, \\
\lambda_3 &> -\sqrt{\lambda_1 \lambda_2}, \\
\lambda_3 + \lambda_4 - |\lambda_5| &> -\sqrt{\lambda_1 \lambda_2}.
\end{aligned} \tag{12}$$

On the experimental side, we consider the following constraints:

(4) The lower mass bound on the charged Higgs bosons: $m_{H^+} > 92$ GeV [9].

(5) The constraints from the LEP search for neutral Higgs bosons. We compute the signals from the Higgsstrahlung production $e^+e^- \rightarrow ZH_i$ ($H_i = h, H$) with $H_i \rightarrow 2b, 2\tau, 4b, 4\tau, 2b2\tau$ [10, 11] and from the associated production $e^+e^- \rightarrow H_i A$ with $H_i A \rightarrow 4b, 4\tau, 2b2\tau, 6b, 6\tau$ [12] and compare them with their LEP data. We also consider the constraints from $e^+e^- \rightarrow ZH_i$ by looking for a peak of M_{H_i} recoil mass distribution of Z -boson [13] and the constraint of $\Gamma(Z \rightarrow H_i A) < 5.8$ MeV when $m_A + m_{H_i} < m_Z$ [14].

(6) The constraints from the LEP search for a light Higgs boson via the Yukawa process $e^+e^- \rightarrow f\bar{f}S$ with $f = b, \tau$ and S denoting a scalar [15]. These constraints can limit $f\bar{f}S$ coupling versus m_S and thus can constrain the parameters of the L2HDM.

(7) The constraints from the W -boson mass. The L2HDM Higgs sector can shift the W -boson mass through radiative corrections. We require the corrected W -boson mass to lie within the 2σ range of the global-fit value. The SM prediction for the W -boson

mass is 80.363 GeV for $m_t = 173$ GeV and $m_H = 111$ GeV [16], and its fitted value is 80.398 ± 0.025 GeV [17]. We use the formula in [18] in calculating the mass and consider the effect of a different top quark mass (in our calculation we take $m_t = 171.3$ GeV). We also subtract the contribution from the SM Higgs boson to avoid double counting the contribution from the Higgs sector.

- (8) The constraints from $Z\tau^+\tau^-$ coupling. For a large $\tan\beta$ the L2HDM Higgs sector can give sizable radiative corrections to $Z\tau^+\tau^-$ coupling. We calculate such corrections and require the corrected $Z\tau^+\tau^-$ coupling to lie within the 2σ range of its fitted value. The SM prediction for this coupling at Z -pole is given by $g_V^{SM} = -0.03712$ and $g_A^{SM} = -0.50127$ [16], and the fitted value given respectively by -0.0366 ± 0.00245 and -0.50204 ± 0.00064 [16]. We use the formula in [18] in our calculation.
- (9) The constraints from τ leptonic decay. We require the L2HDM correction to the branching ratio $Br(\tau \rightarrow e\bar{\nu}_e\nu_\tau)$ to be in the range of $-0.80\% \sim 1.21\%$ [19]. We use the formula in [19] in our calculation.
- (10) The constraints from the muon anomalous magnetic moment a_μ . Now both the theoretical prediction and the experimental measured value of a_μ have reached a remarkable precision, but a significant deviation still exists: $a_\mu^{exp} - a_\mu^{SM} = (29 \pm 8.8) \times 10^{-10}$ [20]. In our analysis we require the L2HDM to account for such difference at 2σ level. Note that in the L2HDM, a_μ gets additional contributions from the one-loop diagrams induced by the Higgs bosons and also from the two-loop Barr-Zee diagrams mediated by A , h and H [21]. If the Higgs bosons are not too light, the contributions from the Barr-Zee diagrams are more important. To account for the discrepancy of a_μ , one needs a light A along with a large $\tan\beta$ to enhance the effects of the Barr-Zee diagram involving the τ -loop. The CP-even Higgs bosons are usually preferred to be heavy since their contribution to a_μ is negative.
- (11) Since the CP-odd Higgs A can be quite light and $h, H \rightarrow AA$ may open up with a large decay width, we require the width of any Higgs boson in the L2HDM to be smaller than its mass (otherwise the Higgs boson may be too fat).

With the above constraints, we scan the parameter space of the L2HDM in the ranges:

$$\begin{aligned}
1 < \tan \beta < 200, \quad 5 \text{ GeV} < m_A < 100 \text{ GeV}, \\
5 \text{ GeV} < m_{h,H} < 350 \text{ GeV}, \quad 92 \text{ GeV} < m_{H^+} < 350 \text{ GeV}, \\
-\sqrt{2}/2 < \sin \alpha < \sqrt{2}/2, \quad |\lambda_5| < 4\pi.
\end{aligned} \tag{13}$$

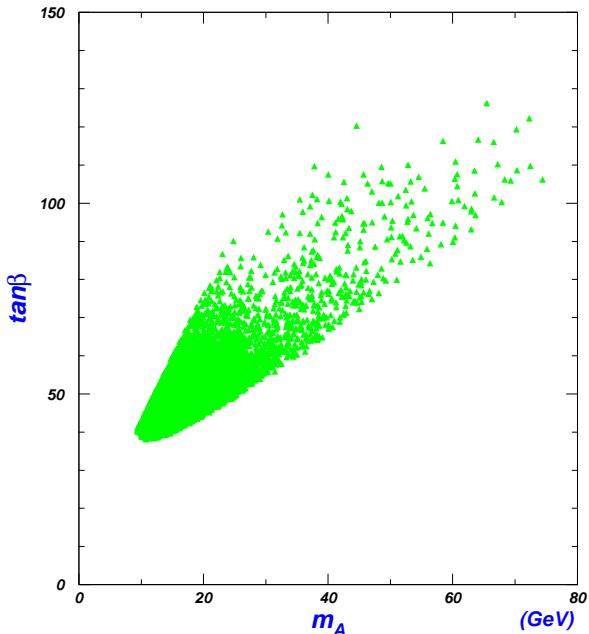


FIG. 1: Scatter plots of the allowed parameter space in the plane of $\tan \beta$ versus m_A .

With 10^{12} random samplings, we get the allowed parameter space shown in Figs.1-2. Fig.1 shows that the allowed parameter space has a light A ($m_A \lesssim 80$ GeV) and a large $\tan \beta$ ($37 \lesssim \tan \beta \lesssim 130$), which mainly comes from the explanation of the a_μ discrepancy. Among the surviving samples displayed in Fig.1, about 95% satisfy $m_A < 30$ GeV and about 70% satisfy $m_A < 20$ GeV, which means that a very light A is highly preferred by the constraints.

Fig.2 shows the allowed parameter space projected in the planes of m_h and m_H versus m_{H^+} . Three characters should be noted about this figure. The first is that all the Higgs bosons are lighter than 350 GeV (lighter than 250 GeV for about 90% of the surviving samples), which is mainly due to the unitary requirement and the a_μ constraint. The second is that h and H can be as light as 58 GeV because the LEP2 bound is relaxed significantly due to the weakened ZZh and ZAH couplings by the sizable mixing angle α and the opening up of the new decay mode $H, h \rightarrow AA \rightarrow 4\tau$ [29]. The third character is that the values of

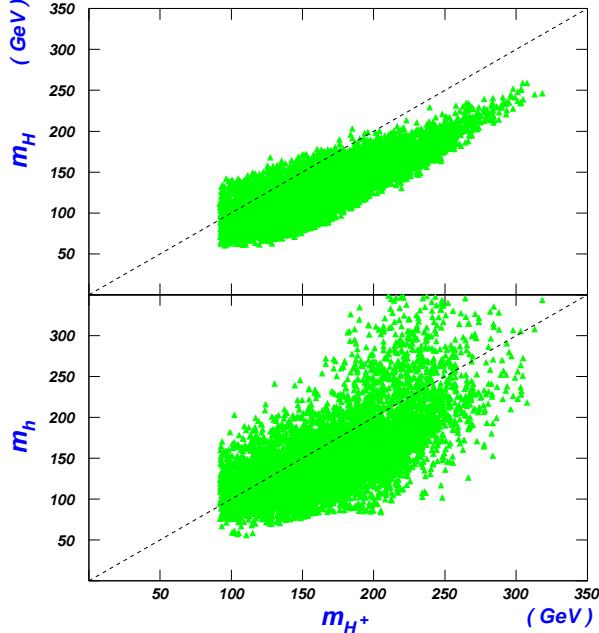


FIG. 2: Same as Fig.1, but projected in the planes of m_h and m_H versus m_{H^+} .

both m_h and m_H are close to the value of m_{H^+} , which is helpful in reducing the L2HDM contribution to the precise electroweak data such as m_W and $Z\bar{\tau}\tau$ couplings at Z-pole. For $m_{H^+} > 250$ GeV, the data require $|\sin(\beta - \alpha)| \sim 1$ [23], and in this case, m_h has little effects on the data so that it can deviate significantly from m_{H^+} .

In summary, the above results indicate that the preferred parameter space of the L2HDM is $37 \lesssim \tan\beta \lesssim 80$, $m_A \lesssim 30$ GeV and the other Higgs bosons lighter than 250 GeV. Note that the above favored region is obtained by considering all the constraints (1-11), instead of any individual constraint. For example, for $\tan\beta > 200$, our results indicate that the CP-odd Higgs boson A as heavy as 120 GeV can still explain a_μ ; but such a large $\tan\beta$ is disfavored by the $Z\bar{\tau}\tau$ coupling at Z-pole or by τ leptonic decay. Another point we should address is that in the L2HDM, the processes $B \rightarrow X_s\gamma$ and $\Upsilon \rightarrow A\gamma$ cannot impose any further constraints [6]. The reason is that in the surviving parameter space, $\tan\beta$ must be larger than 37 and, consequently, the couplings of bottom quark with H^+ and h are suppressed, as shown in Eq.(8). Finally, we would like to emphasize that in contrast to the L2HDM which has a large parameter space to account for the a_μ discrepancy without conflicting with other experimental data, the popular type-II 2HDM is very difficult to do so [21]. This is one of the virtues of the L2HDM.

IV. IMPLICATION ON HIGGS PHENOMENOLOGY

Eq.(8) indicates that the lepton couplings of A , H and H^+ are enhanced by large $\tan\beta$, while quark couplings are suppressed. Since the allowed parameter space has a large $\tan\beta$, the couplings of τ lepton with A , H and H^+ are larger than the top quark couplings. So these scalars will decay dominantly into τ leptons rather than into top quarks (if kinetically allowed). Moreover, a light A can change the phenomenology of other Higgs bosons by opening new decay modes like $h, H \rightarrow AA$, $h, H \rightarrow AZ$ and $H^+ \rightarrow AW^+$. As discussed earlier, in case of a large $\tan\beta$ and a small α , h is the SM-like Higgs boson, mainly responsible for the electroweak symmetry breaking and couples to weak gauge bosons like the SM Higgs. Therefore, the phenomenology of h is of primary importance and will be studied in the following.

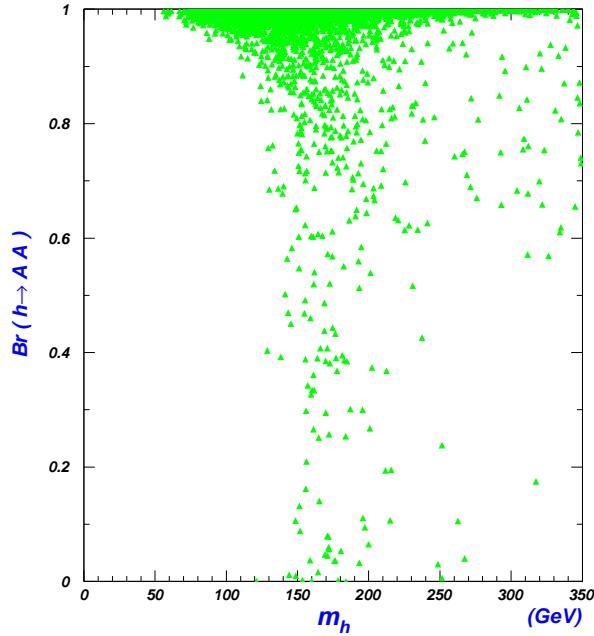


FIG. 3: Same as Fig.1, but showing the branching ratio of $h \rightarrow AA$ versus m_h .

Fig.3 shows the branching ratio of $h \rightarrow AA$ versus m_h . Here we have considered all the decay modes of h including $h \rightarrow VV, AZ, \tau\bar{\tau}, b\bar{b}, t\bar{t}$. This figure shows that for most of the allowed parameter space (about 99%), $h \rightarrow AA \rightarrow \tau\bar{\tau}\tau\bar{\tau}$ is the dominant decay mode. This will make the detection of h difficult at the LHC because the lightness of A (note $m_A < 20$ GeV for about 70% surviving samples) will make the τ leptons from its decay highly collimated [2, 24], which is usually regarded as a difficult scenario in Higgs

discovery at the LHC in the next-to-minimal supersymmetric model [2]. Another reason for the detection difficulty of h is that for more than 80% of the allowed parameter space, the width of h is found to be larger than 10 GeV. Such a wide width will smear the peak of the invariant mass distribution of h -decay products and make the detection more difficult.

We note that in the L2HDM, $A \rightarrow \mu^+ \mu^-$ is the second largest decay mode of A . So $h \rightarrow AA$ can give the multi-muon signal, like the scenario proposed in [25]. Unfortunately, in the L2HDM the branching ratio of $A \rightarrow \mu^+ \mu^-$ is of order 10^{-3} , which will make the channel $h \rightarrow AA \rightarrow 4\mu$ quite hopeless at the LHC. Note that some authors have considered the channels $h \rightarrow AA \rightarrow 2\mu + 2\tau$ [22] and $h \rightarrow AA \rightarrow 4\tau \rightarrow 2\mu + 2\text{ jets}$ [26] as well as the diffractive Higgs production $pp \rightarrow pp + h$ followed by $h \rightarrow 4\tau$ [27] to detect such a h , but all these studies did not consider the worse case of a fat h . We also checked that the branching ratio of $h \rightarrow \gamma\gamma$ is usually suppressed to be less than 10^{-6} and thus too small for the detection.

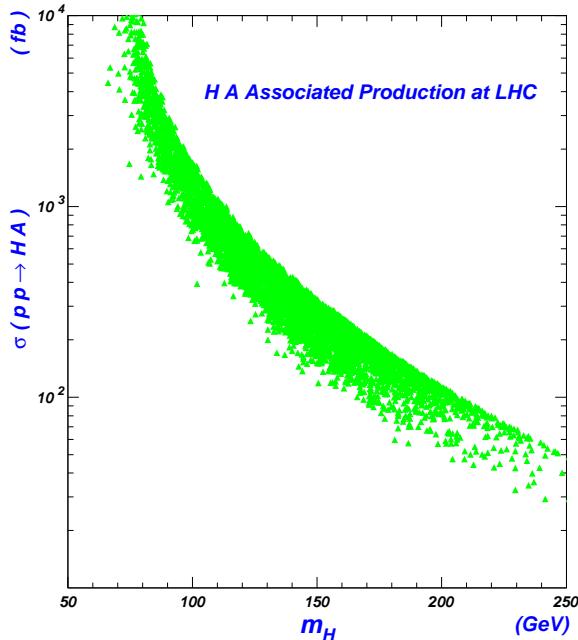


FIG. 4: Same as Fig.1, but showing the cross section of HA associated production at the LHC versus m_H .

Furthermore, we examine other complementary new channels for detecting the Higgs sector of the L2HDM. Firstly, we check the associated HA production at the LHC. The cross section of this process is shown in Fig. 4 and one can learn that for $m_H < 140$ GeV the cross section is larger than 100 fb. The dominant decay of H in this case is found to be

$H \rightarrow AA$, with a branching ratio larger than 80%, so the main signal of this process is 6τ . Due to the lightness of A , great efforts are needed to analyze the signal and the backgrounds in order to detect this process at the LHC. Secondly, we note that A is always lighter than Z boson in the allowed parameter space and thus it may be produced from Z decays. So we investigate the decay $Z \rightarrow \tau\bar{\tau}A$ and find its branching ratio ranging from 10^{-5} to 10^{-4} for $m_A < 40$ GeV (corresponding to 98% of the allowed parameter space). Such a large rate is within the sensitivity of the GigaZ option at the proposed International Linear Collider [28].

V. CONCLUSION

We examined various direct and indirect constraints on the lepton-specific two-Higgs doublet model and then checked the property of the Higgs bosons in the allowed parameter space. We found that the allowed space has a very light CP-odd Higgs boson A ($m_A < 30$ GeV with 95% possibility) which is composed dominantly by the leptonic Higgs and decays dominantly into $\tau^+\tau^-$. The SM-like Higgs boson h decays dominantly in the mode $h \rightarrow AA \rightarrow 4\tau$, which may make the Higgs discovery difficult at the LHC. We also checked other possibilities for testing the Higgs sector of this model and found that the decay $Z \rightarrow \tau^+\tau^-A$ has a branching ratio ranging from 10^{-5} to 10^{-4} , which may be accessible at the the GigaZ option of the ILC.

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